Graviton mass and cosmological constant: a toy model

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Abstract

I consider a simple model where the graviton mass and the cosmological constant depend on a scalar field with appropriate couplings and I calculate the graviton propagator and the resulting effective action for the scalar field.

1 Introduction

Infrared modifications of gravity with a small graviton mass appear in various models in cosmology and particle physics [1] and it is interesting to investigate their consistency. A first problem that appears in these models is the well known van Dam-Veltman-Zakharov (vDVZ) discontinuity that persists for arbitrarily small graviton mass [2, 3]. It has been shown that it is possible to circumvent this problem in the presence of a (positive or negative) cosmological constant [4, 5, 6], then, however, another problem of consistency appears: in the case of a positive cosmological constant, in particular for de Sitter space-time, one encounters the Higuchi instability for $m^2 < 2H^2$, where m is the graviton mass and H the Hubble expansion rate [6]. It is argued that a scalar degree of freedom propagates then with the wrong sign of the kinetic term, a ghost, and several aspects of this problem have been investigated [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

The problem of spontaneous symmetry breaking in order to give a mass to the graviton has also been considered [19, 20, 21, 22, 23], which is more involved than the usual process of symmetry breaking in quantum field theory, since a massive graviton has five degrees of freedom compared to the two degrees of freedom for an ordinary, massless graviton.

In order to gain some more perspective on these problems I consider here a simple toy model, where both the graviton mass and the cosmological constant terms depend on (the same) scalar field, ϕ. As far as the cosmological constant term is concerned this is just an ordinary mass term for ϕ. The graviton mass term is added by hand since there is no non-linear completion of massive gravity.

2 Graviton propagator and effective action

In the usual Einstein-Hilbert action for gravity I add a scalar field, ϕ, with a mass term

$$S_1 = \int d^4x \, \sqrt{-g} \left(-\frac{R}{16\pi G} - \frac{1}{2}m^2\phi^2 \right) \tag{1}$$

as well as a graviton mass term

$$S_2 = -\int d^4x \, \frac{1}{2}\alpha\phi^2(h_{\mu\nu}^2 - h^2) \tag{2}$$

at quadratic order after linearization around a flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu} \tag{3}$$

where $\kappa^2 = 32\pi G = \frac{32\pi}{M_P^2}$, $h = h_\mu^\mu$ and α a dimensionless coupling.

Although the action S_1 is invariant with respect to the gauge transformation

$$\delta h_{\mu\nu} = \kappa(\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu), \tag{4}$$

corresponding to a massless graviton with two degrees of freedom, the addition of S_2 breaks this invariance and describes a massive graviton with five degrees of freedom and mass $m_g^2 = \alpha\phi^2$.

If the graviton mass term is considered as an interaction, as is done here, one still needs to gauge fix the action S_1 with the Stueckelberg procedure, where functional integration over ϵ_μ is done, in order to introduce a gauge fixing term. This will generate additional terms coming from the non-invariant S_2 which I will describe later.

Adding the gauge fixing term $(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h)^2$ the quadratic terms of S_1 give the lagrangian

$$L_1 = -\frac{1}{2}h_{\mu\nu}\,\partial^2\,K^{\mu\nu,\beta\rho}h_{\beta\rho} - \frac{1}{2}\lambda\phi^2\left(h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}h^2\right) \tag{5}$$

where $\lambda = \frac{8\pi m^2}{M_P^2}$ and

$$K^{\mu\nu,\beta\rho} = \frac{1}{2}(\eta^{\mu\beta}\eta^{\nu\rho} + \eta^{\mu\rho}\eta^{\nu\beta}) - \frac{1}{2}\eta^{\mu\nu}\eta^{\beta\rho} \tag{6}$$

or, more compactly,

$$K = I - \frac{1}{2}gg. \tag{7}$$

Since $K^2 = I$, the inversion of the quadratic terms in order to get the bare graviton propagator is straightforward,

$$G = \frac{i}{k^2}(I - \frac{1}{2}gg), \tag{8}$$

as is the inclusion of the "cosmological constant" interaction term, $-i\lambda\phi^2(I - \frac{1}{2}gg)$, to get

$$G_{(\lambda)} = \frac{i}{k^2 - \lambda\phi^2}(I - \frac{1}{2}gg) \tag{9}$$

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(where k is the graviton momentum).

The resummation of the interaction corresponding to the graviton mass, $-i\alpha\phi^2(I - gg)$, can also be done (Fig. 1), yielding the final dressed graviton propagator

$$\tilde{G} = \frac{i}{k^2 - \lambda\phi^2 - \alpha\phi^2}(I - \frac{1}{4}gg) - \frac{i}{k^2 - \lambda\phi^2 - 3\alpha\phi^2}(\frac{1}{4}gg). \qquad (10)$$

It is interesting to see that, even in the limit of $\lambda = 0$, and writing $m_g^2 = \alpha\phi^2$ for the graviton mass, one gets for the massive graviton propagator

$$G_{m_g} = \frac{i}{k^2 - m_g^2}(I - \frac{1}{4}gg) - \frac{i}{k^2 - 3m_g^2}(\frac{1}{4}gg), \qquad (11)$$

a different expression than what is normally used, that does not suffer from the vDVZ discontinuity. Of course it is well known that when this is absent there are the problems of ghost-like states [24], and indeed, one can see that in (11) there are five degrees of freedom corresponding to a massive graviton, plus two longitudinal degrees of freedom, one normal and one ghost-like, with poles at different positions that do not cancel. This would create consistency problems for the full theory. At this order, however, one can calculate the effective action for the scalar field using the dressed propagator, $\tilde{G}$ from (10), to get

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left(\frac{1}{2}Z(\phi)(\partial\phi)^2 - U(\phi)\right) \qquad (12)$$

with

$$U(\phi) = \frac{1}{2}m^2\phi^2 - \frac{m^2\phi^4}{4\sigma^2} + \frac{\lambda(2\lambda + 3\alpha)}{16\pi^2}\phi^4 \left(\ln\frac{\phi^2}{\sigma^2} - \frac{1}{2}\right) \qquad (13)$$

the effective potential, with renormalization conditions that describe a second minimum at $\phi = \sigma$, and

$$Z(\phi) = \frac{1}{3(4\pi)^2} \left(9(\lambda + \alpha) + \frac{(3\alpha + 2\lambda)^2}{3\alpha + \lambda}\right) \qquad (14)$$

(independent of ϕ at this order).

Before discussing this effective action (for the canonically normalized field) I should comment on the higher quantum gravitational corrections, the corrections that arise from expanding around flat space-time, as well as

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the additional terms that arise from the Stueckelberg procedure: first of all there are the graviton loops like the first diagram of Fig. 2 that include the graviton self-interaction vertex that is of order $\kappa^2 p^2$, with p the graviton momentum, and which are non-renormalizable. If one treats quantum gravity as an effective theory [25] these generate additional corrections to the effective action with coefficients that, from power counting, turn out to be of order ϕ/M_P. Second, the expansion around flat space-time is consistent if there is no cosmological constant term (strictly at $\phi = 0$). For non-zero ϕ (or U) the expansion around flat space-time introduces tadpole terms like those in the second diagram of Fig. 2, which are also suppressed by powers of ϕ/M_P. The effective action derived, therefore, has the advantage of coming from renormalizable terms, it is inconsistent, however, when $\phi \sim M_P$, where quantum gravitational corrections take over, as expected.

Finally, when integrating over the gauge parameter, ϵ_μ, in order to introduce the gauge fixing term, one gets additional terms because of the non-invariance of S_2:

$$\delta L = \alpha \kappa^2 \phi^2 \left(\epsilon_\mu \partial^2 \epsilon^\mu + (\partial \epsilon)^2 + 2h(\partial \cdot \epsilon) \right) \tag{15}$$

(this expression has been derived for constant ϕ and the gauge condition $\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h$ has been used). The integration over ϵ_μ can now be done using an additional gauge fixing for the abelian gauge invariance of the last expression, leading to a non-local, ϕ-dependent term. For $\alpha\phi^2 << M_P^2$ the variation of this term can be neglected (and when the gauge condition $\partial \cdot \epsilon = 0$ is used these terms will decouple from the gravitons to this order).

As far as the effective action (12) for the canonically normalized field ϕ is concerned, we can see that, if we require the second minimum, σ, to correspond to $U(\sigma)$ equal to zero or negative we need σ to be of order M_P, which is where our approximations break down. So, to this order, one cannot start with a flat space-time, with a massless graviton, at $\phi = 0$, and generate a theory with a massive graviton (as expected). It is possible, however, to have a second, relative minimum at $\phi = \sigma << M_P$, with positive $U(\sigma)$, corresponding to a metastable vacuum with massive graviton and finite cosmological constant that decays to flat space-time with a massless graviton.

3   Comments

In this work I used a simple model where the graviton mass (as well as the cosmological constant) are obtained through dimensionless (renormalizable) couplings of a scalar field, ϕ with the graviton field. I obtained the effective action for ϕ and showed the limits of its applicability so that quantum gravity effects do not take over. One cannot obtain symmetry breaking from massless graviton to massive graviton, as is expected, but it is possible to generate a metastable vacuum with a massive graviton that decays to flat space-time with a massless graviton.

As far as extensions of this model are concerned it is possible to add a small cosmological term to the tree level action and consider corrections similarly to the work of [26], similar conclusions will hold provided the corrections are within the limits of the approximations.

It would also be interesting to see if the conditions for an inflationary model can be satisfied for some values of the parameters; it is obvious that the limits of the approximation break down when chaotic inflationary models are considered, it is possible, however, to modify the present model using two different scalar fields, similarly to a hybrid model, and check the parameter space of the full effective action.

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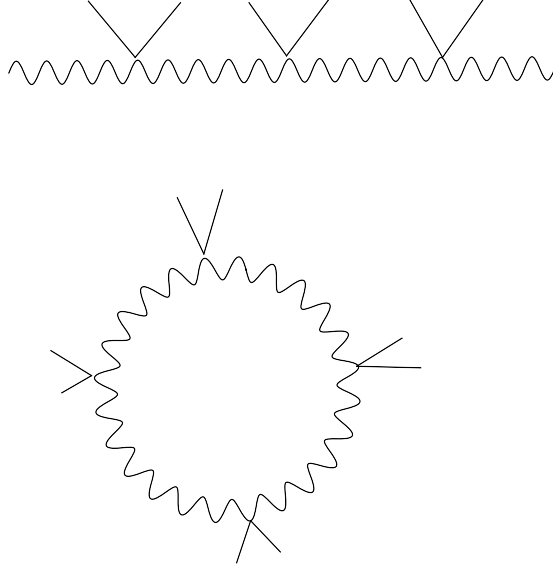[27] D. Binosi and L. Theussl, *Comput. Phys. Commun.*, **161**, 76 (2004).

Fig. 1: The resummation of the graviton propagator (curly line) with the interactions with the scalar field (solid line), and the loops considered for the calculation of the effective action.
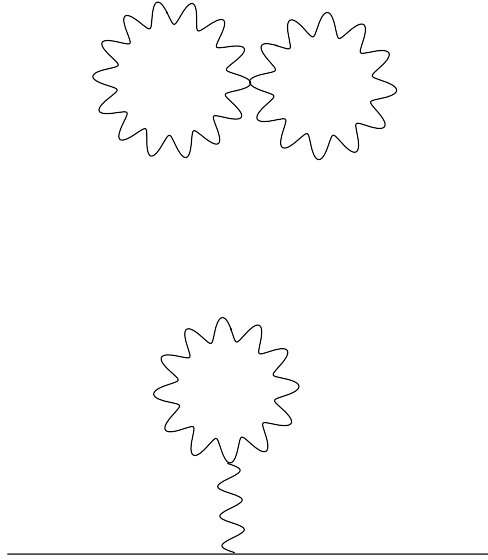
Fig. 2: Some diagrams that are neglected from quantum gravity corrections and tadpole contributions.